Production Optimization Using Derivative Free Methods Applied to Brugge Field

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Outline

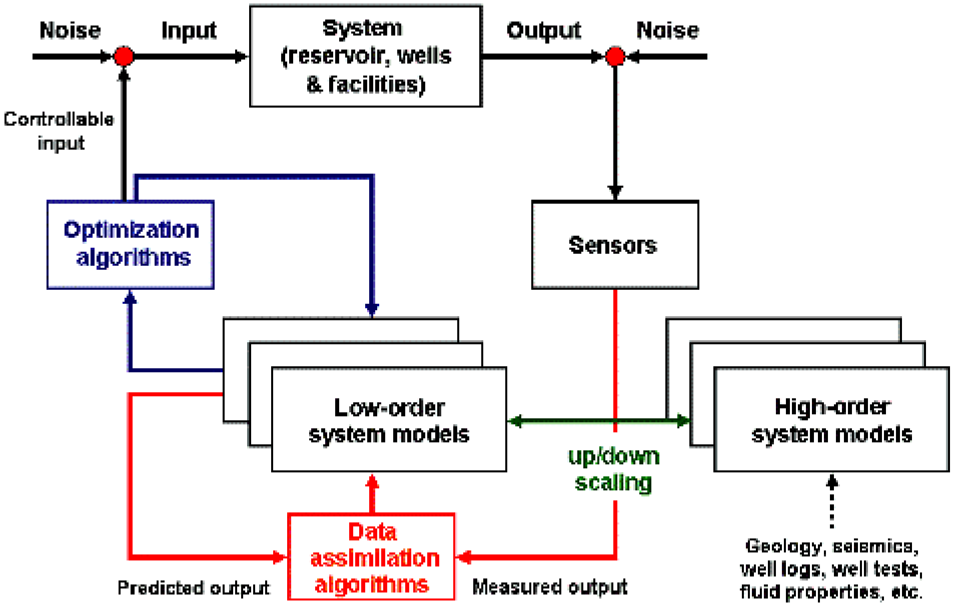
- Closed-Loop Reservoir Management
- Derivative Free Optimization
  - Nelder-Mead
  - Hooke-Jeeves
  - Particle Swarm Optimization
- Brugge Field
- Optimization Problem
- Results
- Conclusions

Closed-loop Reservoir Management

- Reservoir Management:

  using the geological and engineering information to predict and optimize the future production of oil and gas reservoirs.

- CLRM: Combination of (SPE 119098, Jansen et al. 2009)
  - model-based optimization;
  - data assimilation (SPE 119101, Lorentzen et al. 2009).



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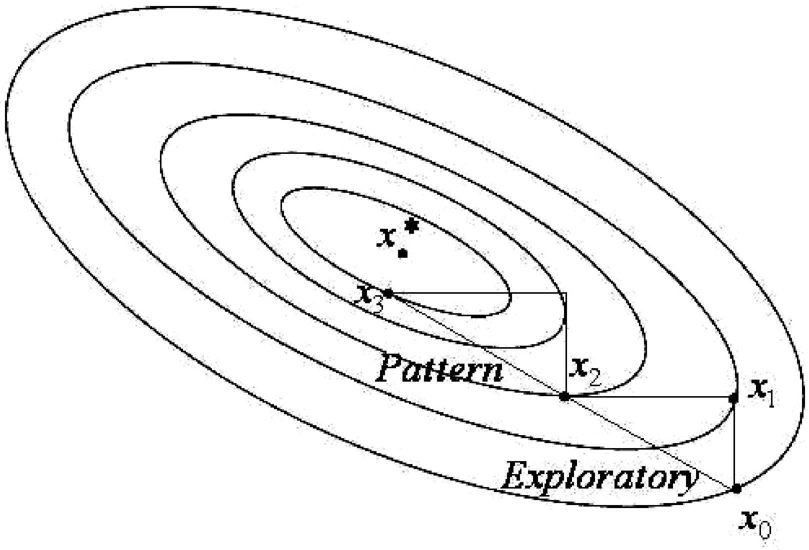
Derivative Free Optimization

- The methods use the function values at a set of points;
- Advantage: do not calculate the gradients;
- Disadvantage: are not well developed for the constraints;
- Types:
  - Pattern search (deterministic);
  - Simplex reflection (deterministic);
  - Random search (stochastic).

Hooke-Jeeves (Pattern Search)

- Two main steps
  - Exploratory search
  - Pattern search
- Inputs
  - Initial point, x^{k-1} ;
  - Perturbation size, Δx;
  - Step reduction factor, q;
  - Convergence tolerance, ε .

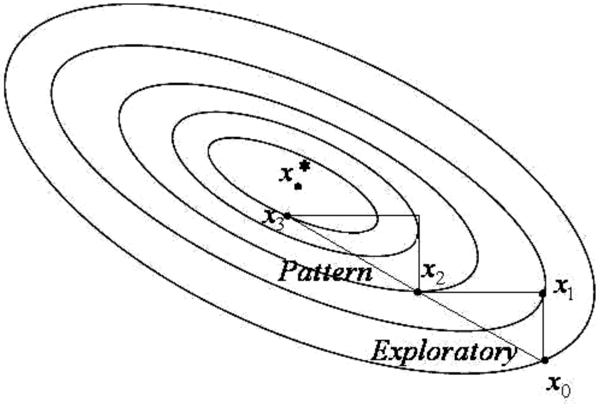


Hooke-Jeeves

- Procedure
  1. Function evaluation at initial point, $f(x^{k-1})$;
  2. Type I exploratory search:
     1. Perturbation, $x^k=x^{k-1} \pm \Delta x$, and movement, if the objective function value is improved;
     2. Repeating for all variables;
  3. Evaluation of exploratory search success, comparing the function values at x^{k-1} and x^k:
     1. If failed: check for termination:
        - Yes: stop;
        - No: $\Delta x = \Delta x/q$ and return to 2;
     2. If successful: continue;

Hooke-Jeeves

4. Pattern move: $x_p^{k+1} = 2x^k - x^{k-1}$
5. Type II exploratory search, x^{k+1};
6. Evaluation of pattern search success, comparing the function values at x^k and x^{k+1}:
    1. If failed: restore the value to the step before pattern search and start from step 2;
    2. If successful: set $x^{k-1}=x^k$, $x^k=x^{k+1}$ go to step 4.



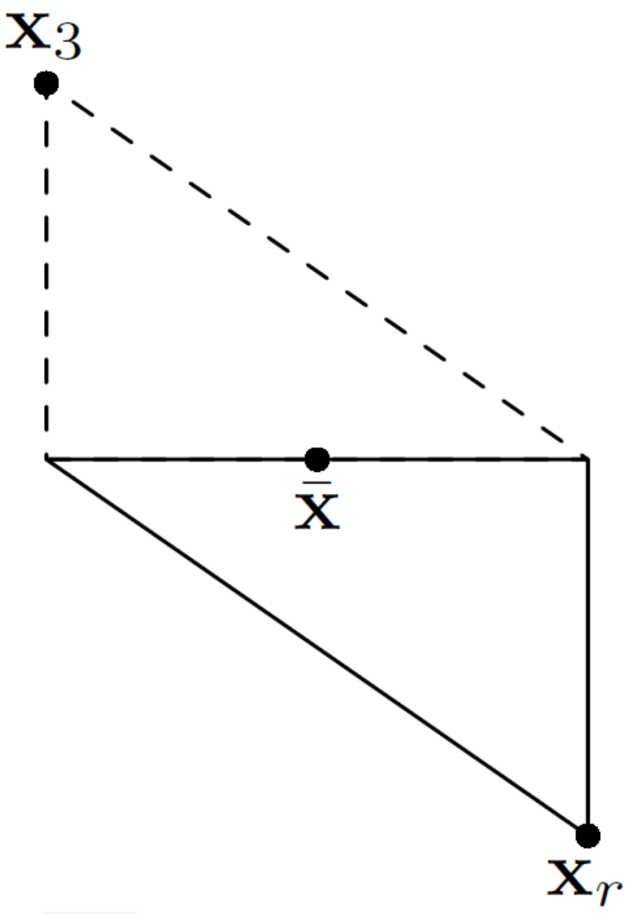
Nelder-Mead (Reflection Simplex)

- Minimization of a function of n variables by:
  - Comparing the function values at the n+1 vertices of a simplex;
  - Replacement of the highest value by a new point.
- Definitions:
  - $P_0, P_1, \ldots, P_n$: n+1 vertices of a simplex;
  - f_i: the function value at P_i;
  - f_h: the highest function value;
  - f_l: the lowest function value;
  - P’: centroid of the simplex excluding f_h;

Nelder-Mead

1. Reflection
   - $P^{*}=(1+\alpha)P' - \alpha P_h$;
   - P^{*}: reflection of P_h;
   - α : reflection coefficient,
     - positive constant;
     - universally 1;

- If $f_l<f^{*}<f_h$: then P_h is replaced by P* and the algorithm restarts from reflection.
- If $f^{*}<f_l$: expansion (step 2);
- If $f_i<f^{*}$, $i \neq h$: contraction (step 3).



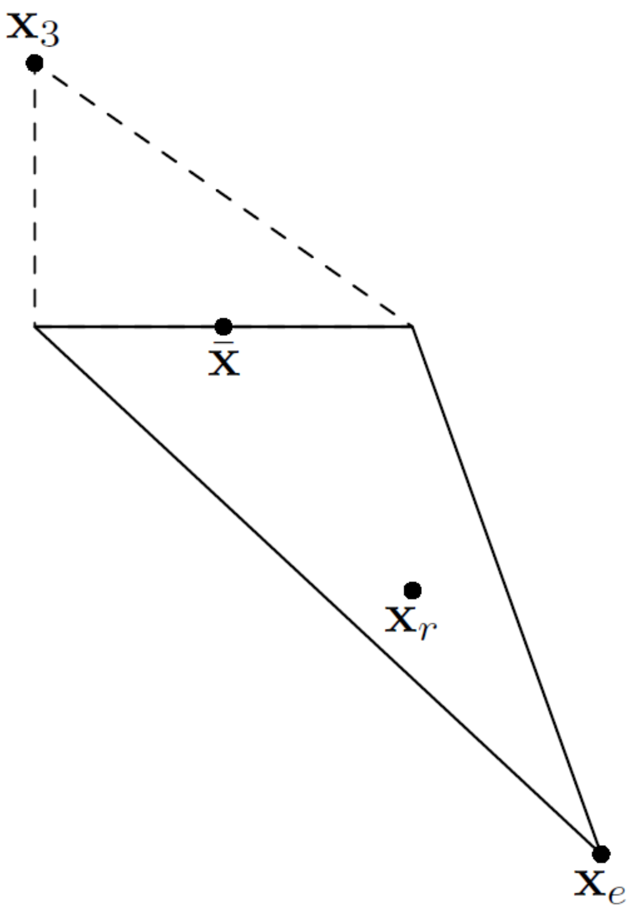
Nelder-Mead

- If $f^* < f_l$: a new minimum has been produced.

2. Expansion
   - $P^{**} = \gamma P^* + (1-\gamma)P'$;
   - P^{**}: expansion of P^*;
   - γ: expansion coefficient;
     - Larger than unity;
     - Universally 2.

- If $f^{**} < f^*$: P_h is replaced by P^{**};
- If $f^{**} > f^*$: the expansion is failed and P_h is replaced by P^*;

The procedure is restarted from reflection.



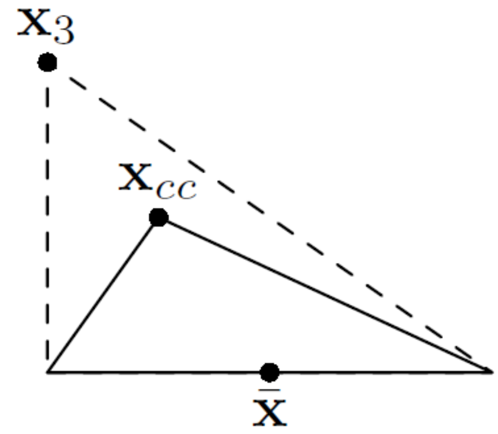
Nelder-Mead

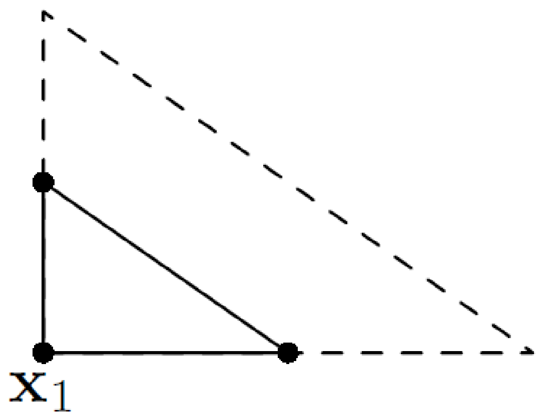
- $f_i < f^*, i \neq h$:

3. Contraction
   - $P^{**} = \beta P_h + (1-\beta)P'$;

- Inside contraction
   - P**: contraction of P*;
   - β : contraction coefficient;
   - Between zero and unity;
   - Universally 0.5.
- P_h is replaced by P**, unless:
- $f^{**} > \min(f_h, f^*)$ then:
   - Shrink: $P_i = (P_i + P_l)/2$;

The algorithm is restarted.



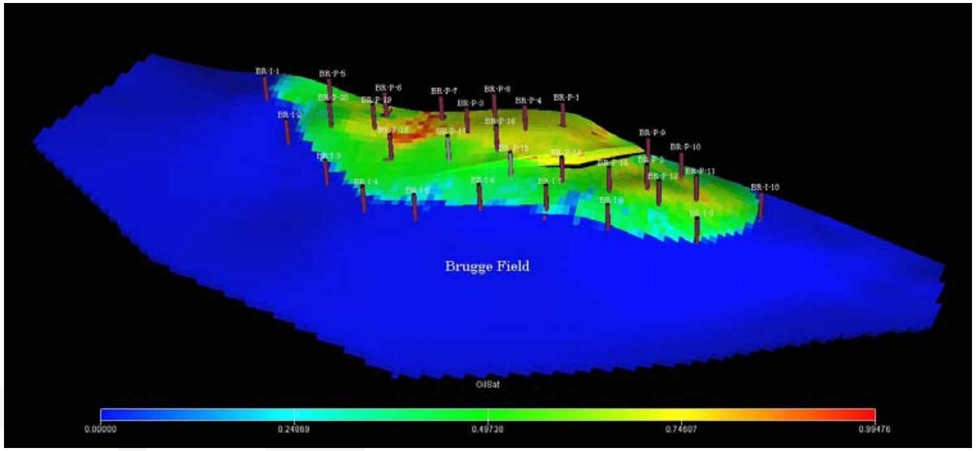


Particle Swarm Optimization (PSO)

- Global search
- Continuous non-linear functions
- Procedure
  - Initialize by a random population (i particles)
  - Each particle: n variables;
  - Update:
    - $x_{i,\,k+1} = x_{i,\,k} + v_{i,\,k+1}$;
    - $v_{i,\,k+1} = v_{i,\,k} + c_1 r_1 (p_{i,\,k} - x_{i,\,k}) + c_2 r_2 (p_{g,\,k} - x_{i,\,k})$;

Brugge Field

- Typical North Sea Brent type field
  - Total grid blocks: 60,048
  - Active grid blocks: 45,550
- 20 producers (crestal)
- 10 injectors (pripheral)
- Three completions in each well
- 10 years history match
- 20 years production
- Open completions
  - Injectors: 10x3
  - Producers: 20x3-6=54
  - Total: 84
- 104 realizations
  - All history matched;
  - A mean model was prepared.



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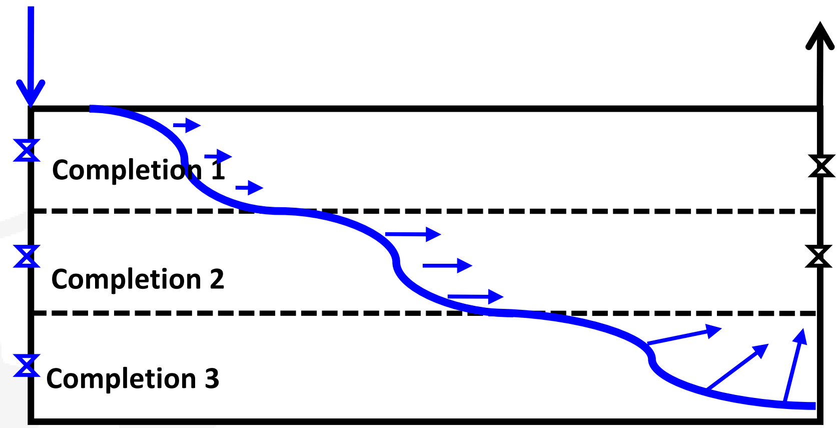
Formulation

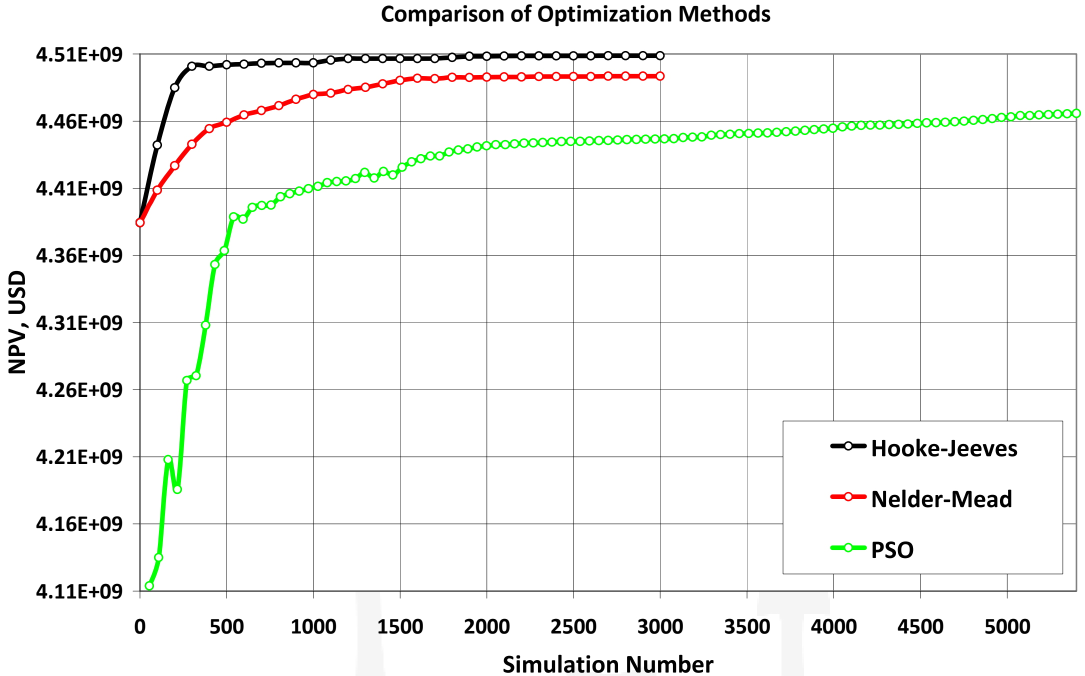
- Objective function: NPV
- Optimization variables
  - Shut-in water cuts
- Optimization constraints
  - 0<WCT<0.94;
  - BHP_p>725 psi;
  - WLPR<3000 bbl/day;
  - BHP_i<2610 psi;
  - WWIR<4000 bbl/day;

$$NPV = \sum_{k=1}^{M} \Delta t \frac{Q_o r_o - Q_w r_w - Q_i r_i}{(1+b/100)^{t_k}}$$

Parameter	Value
Oil Price ($)	80
Water Opration Cost ($)	5
Water Injection Cost ($)	5
Discount Rate (percent per year)	10

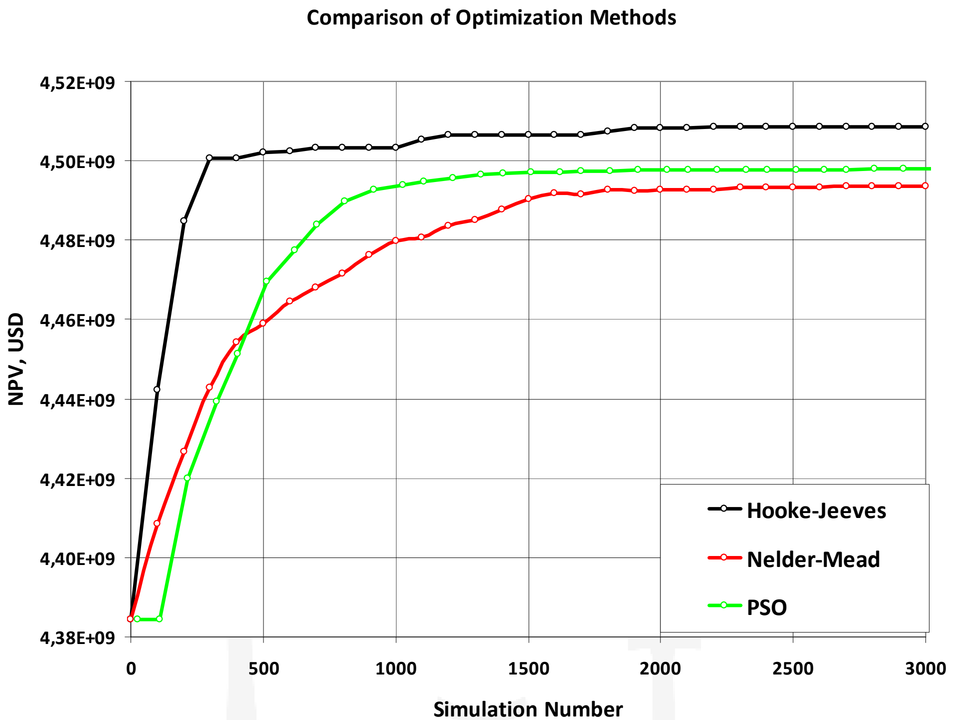


Results



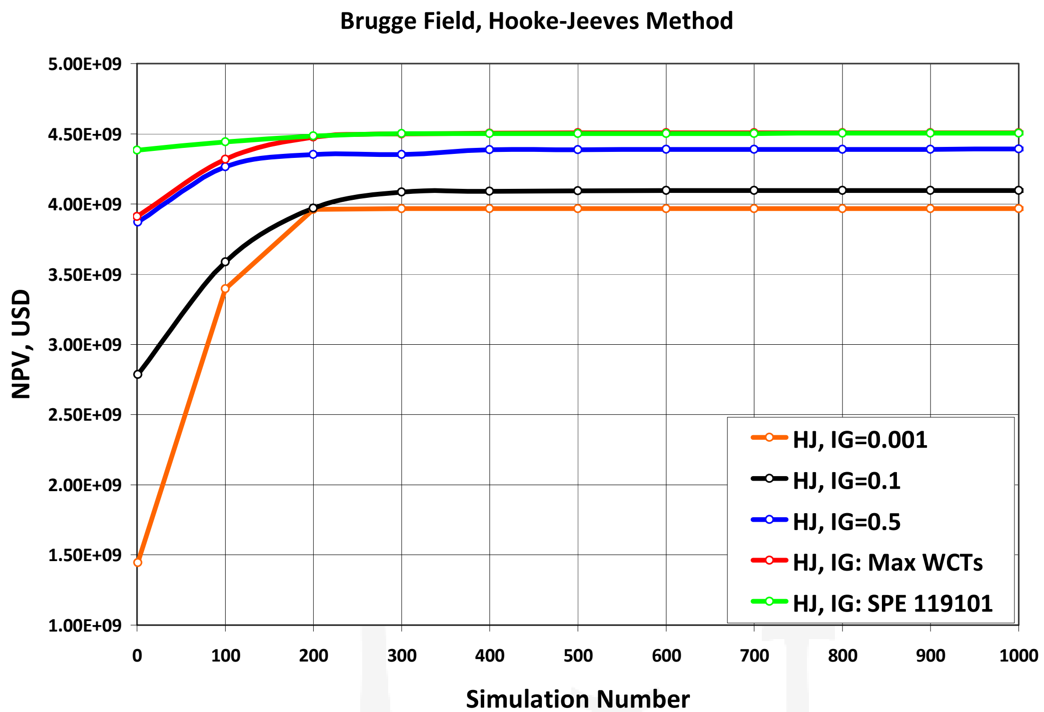
Optimization Method	Optimized Objective Value (MMM$)
Hooke-Jeeves	4.51
Nelder-Mead	4.49
Particle Swarm Optimization	4.47

Results



Optimization Method	Optimized Objective Value (MMM$)
Hooke-Jeeves	4.509
Nelder-Mead	4.493
Particle Swarm Optimization	4.499

HJ against Initial Solution



Initial Solution	Initial Function Value, $	Optimal Function Value, $
0.001	1.44E+09	3.97E+09
0.1	2.79E+09	4.10E+09
0.5	3.87E+09	4.39E+09
Max WCT Limits	3.91E+09	4.5100E+09
SPE 119101	4.38E+09	4.5087E+09

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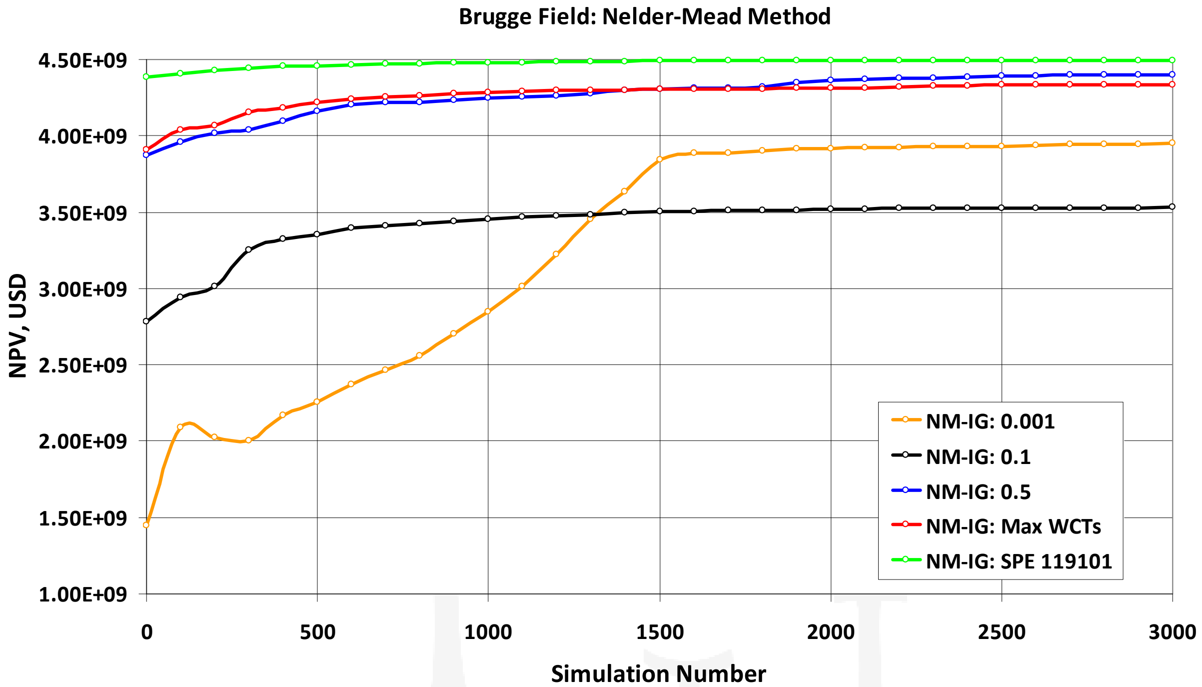
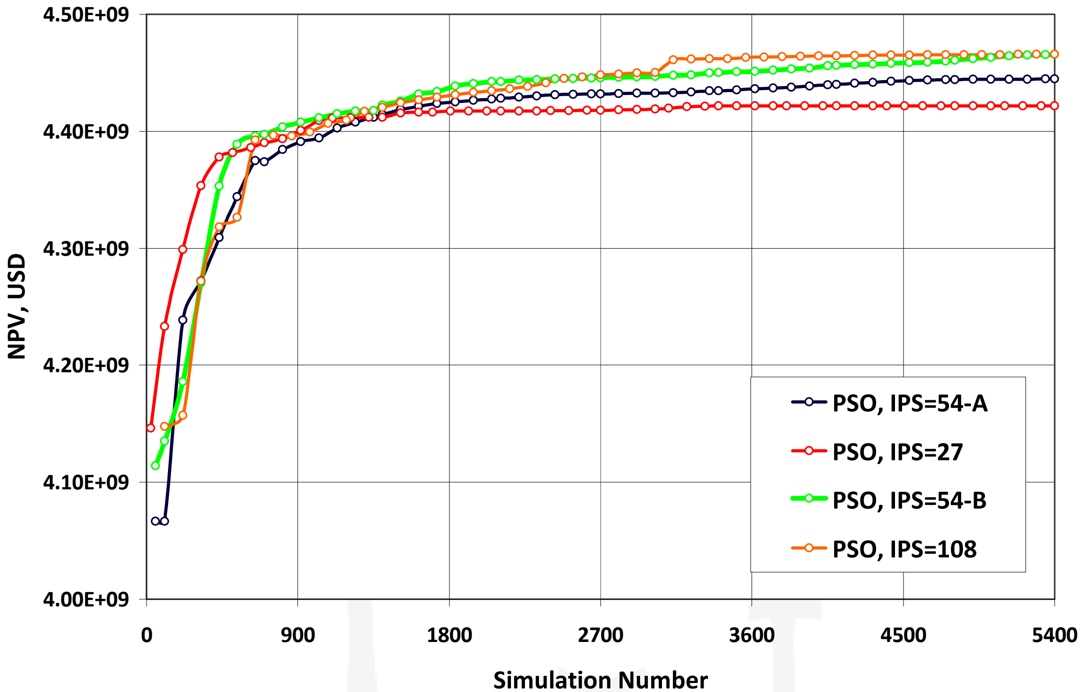
NM against Initial Solution



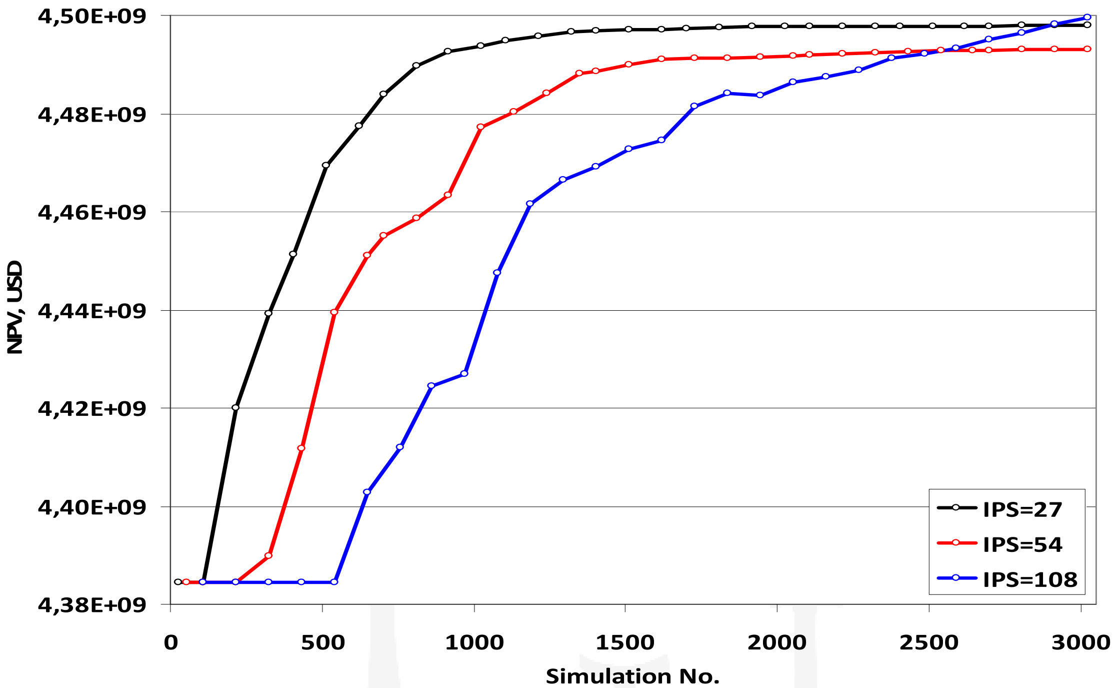
Initial Solution	Initial Function Value	Optimal Function Value
0.001	1.44E+09	3.95E+09
0.1	2.79E+09	3.53E+09
0.5	3.87E+09	4.40E+09
Max WCT Limits	3.91E+09	4.34E+09
SPE 119101	4.38E+09	4.49E+09

PSO against Initial Solution



Initial Population Size	Initial Function Value	Optimal Function Value
27	4.15E+09	4.42E+09
54-A	4.07E+09	4.44E+09
54-B	4.11E+09	4.46E+09
108	4.15E+09	4.47E+09

PSO including Initial Guess



Initial Population Size	Initial Function Value, $	Optimal Function Value, $
27	4.38E+09	4.498E+09
54	4.38E+09	4.493E+09
108	4.38E+09	4.499E+09

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Conclusions

- HJ
  - More efficient;
  - Highest objective value (good initial guess);
  - Dependent to initial solution (deterministic).
- NM
  - Slow rate of convergence;
  - Reasonable optimal objective value: when it is closer to optimum;
- PSO
  - High objective value: independent of initial guess (stochastic);
  - Including a good initial guess:
    - Improves the final optimal objective value;
    - Faster rate of convergence;

Thanks for your attention!

As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality.
Albert Einstein

Questions?