Oppdatering av reservoar modeller med ensemble Kalman filter

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IRIS

Outline

- Kalman filter
- Ensemble Kalman filter
- Updating reservoir models
- Challenges with ensemble Kalman filter
- Summary

The Kalman filter

Kalman (1960): Recursive solution to the discrete data linear filtering problem.

Process: $x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1}$

Measurement: $z_k = Hx_k + v_k$

$w_{k-1} \sim N(0, Q)$ process noise

$v_k \sim N(0, R)$ measurement noise

Apriori estimate of error covariance $P_k^f = E\left((x_k - \hat{x}_k^f)(x_k - \hat{x}_k^f)^T\right)$

The Kalman filter - II

$$\hat{x}_k^a = \hat{x}_k^f + K(z_k - H\hat{x}_k^f)$$

where the Kalman gain

$$K = P_k^f H^T (H P_k^f H^T + R)^{-1}$$

is obtained by minimizing aposteriori error covariance

$$\begin{aligned} P_k^a &= E\left((x_k - \hat{x}_k^a)(x_k - \hat{x}_k^a)^T\right) \\ &= E\left(x_k - (\hat{x}_k^f + K(z_k - H\hat{x}_k^f)))(x_k - (\hat{x}_k^f + K(z_k - H\hat{x}_k^f)))^T\right) \end{aligned}$$

with respect to K

Kalman filter - Summary

Kalman filter - time update equations

$$\hat{x}_k^f = A\hat{x}_{k-1}^a + Bu_{k-1}$$
$$P_k^f = AP_{k-1}^a A^T + Q \qquad (w_k \sim N(0, Q))$$

Kalman filter - measurement update equations

$$K_k = P_k^f H^T (HP_k^f H^T + R)^{-1} \qquad (v_k \sim N(0, R))$$
$$\hat{x}_k^a = \hat{x}_k^f + K_k(z_k - H\hat{x}_k^f)$$
$$P_k^a = (I - K_k H)P_k^f$$

Kalman filter - Summary

Kalman filter - time update equations

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$$P_k^a = (I - K_k H)P_k^f$$

$p(x_k|z_k) \sim N(\hat{x}_k^a, P_k^a)$

Kalman filter - Summary

Kalman filter - time update equations

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Kalman filter - measurement update equations

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$$\hat{x}_k^a = \hat{x}_k^f + K_k(z_k - H\hat{x}_k^f)$$
$$P_k^a = (I - K_k H)P_k^f$$

$p(x_k|z_k) \sim N(\hat{x}_k^a, P_k^a)$

Non-linear generalization: Extended Kalman filter.

Ensemble Kalman filter - I

Evensen, 1994

Model:

$$
\begin{aligned}
x_k &= f(x_{k-1}, u_{k-1}) + w_{k-1} \\
z_k &= H x_k + v_k
\end{aligned}
$$

Use an ensemble of realizations of the model:

$$\left[x_{k,1} x_{k,2} \ldots x_{k,N} \right]$$

Compute statistics from ensemble:

$$\hat{x}_k^* = \frac{1}{N} \sum_{i=1}^{N} x_{k,i}, \quad P_k^* = \frac{1}{N-1} \sum_{i=1}^{N} (x_{k,i} - \hat{x}_k^*)(x_{k,i} - \hat{x}_k^*)^T$$

Ensemble Kalman filter -II

Ensemble Kalman filter - time update equations

$$x^f_{k,i} = f(x^a_{k-1,i}, u_{k-1}) + w_{k,i}$$
$$w_{k,i} \sim N(0, Q)$$

Ensemble Kalman filter - measurement update equations

$$z_{k,i} = z_k + v_{k,i}$$
$$v_{k,i} \sim N(0, R)$$
$$K_k = P^f_k H^T (H P^f_k H^T + R)^{-1}$$
$$x^a_{k,i} = x^f_{k,i} + K_k(z_{k,i} - H x^f_{k,i}))$$
$$(P^a_k = (I - K_k H) P^f_k)$$

Ensemble Kalman filter - III

Applications of ensemble Kalman filter:

- oceanographic models
- atmospheric models
- meteorology
- hydrology
- fish stock assessment
- ....
- petroleum

Continuous model updating

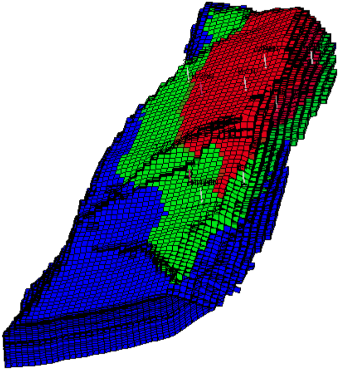
The Kalman filter was originally intended for updating the states of the model. In petroleum science many of the models has poorly known model parameters.

- Need also to tune model parameters!
- Uncertainty in initial conditions are often ignored.
- Application areas:
  - wellflow models (Lorentzen et. al. 2001)
  - reservoir models (Nævdal et. al. 2002, Haugen et. al. 2008)

Reservoir simulation

- Three phase reservoir flow (oil, water, gas)
    - Darcy equation
    - Mass conservation
    - Boundary: Well flow
- Dynamic variables: Pressure, saturations, dissolved gas, vaporized oil
- Update porosity and permeability in each grid cell (++)
- Simulated with commercial reservoir simulator

A North Sea Reservoir
Grid size: $45 \times 75 \times 26$
47795 active cells
$O(3 \cdot 10^5)$ variables



Initial ensemble

Based on prior geostatistical model

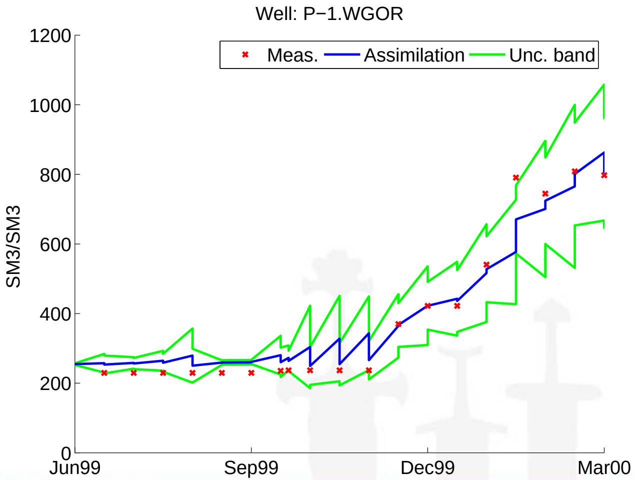
- Horizontal correlation length for each layer
  - Gaussian variogram used in example
- Vertical correlation between the layers
- Correlation between porosity and permeability

Measurement uncertainties

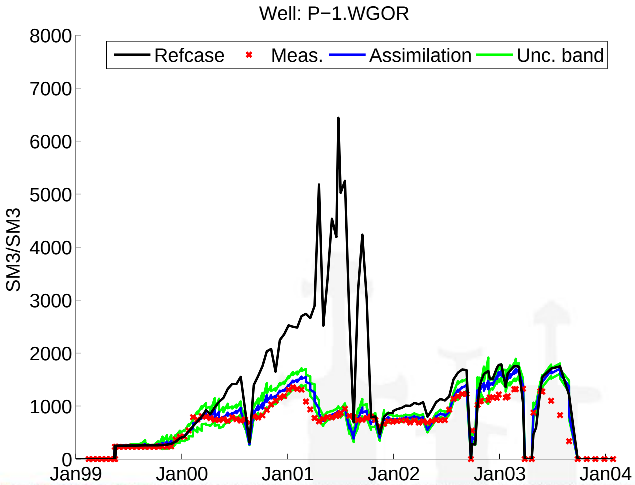
Quantifying these are difficult!
For particular example:

- Oil production rate: 15%
- Water cut: 15%
- Gas-oil ratio: 15%
- Bottomhole pressure: 10%

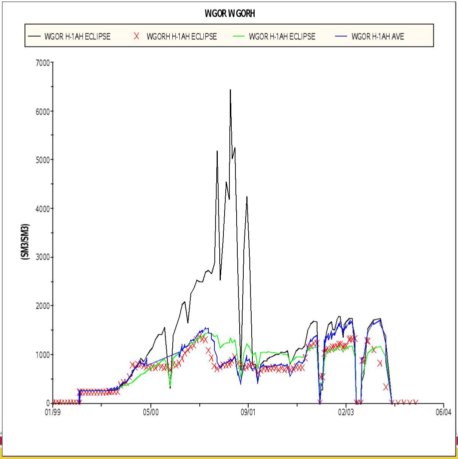
Evolvement of EnKF



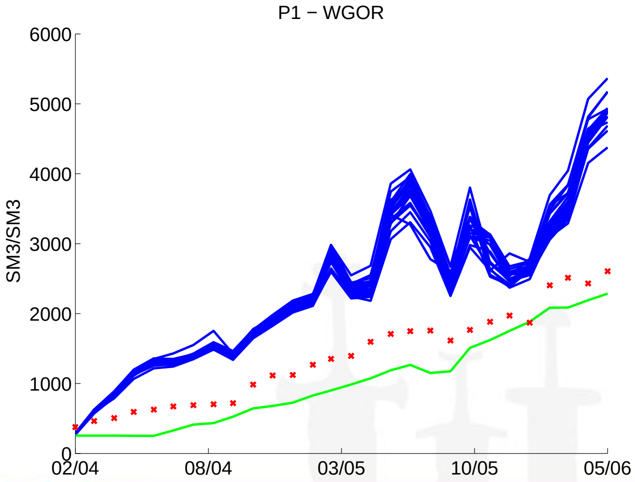
Evolvement of EnKF



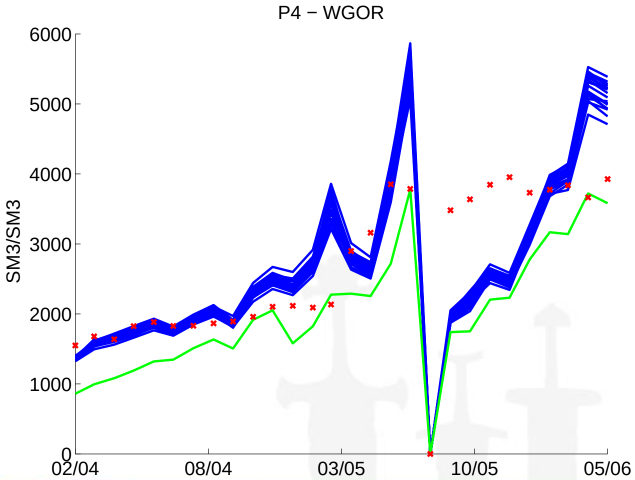
Comparison with manual history match



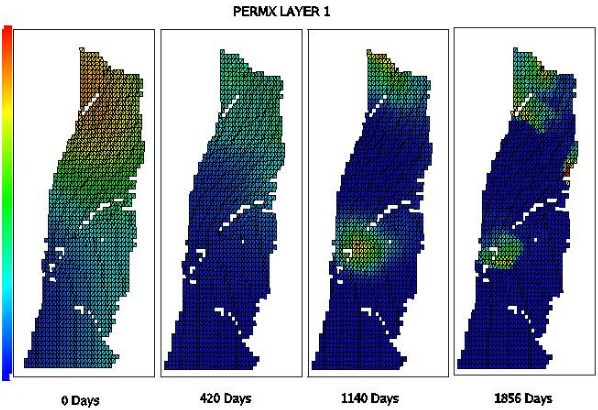
Forecast with updated model



Forecast with updated model



Estimated permeability field



Challenges with ensemble Kalman filter

- Difficult to estimate large-scale covariance matrix
  - $P_k^* = \frac{1}{N-1} \sum_{i=1}^{N} (x_{k,i} - \hat{x}_k^*)(x_{k,i} - \hat{x}_k^*)^T$
- Non-linearity & non-Gaussian distributions
  - Reservoir applications:
    - Gaussian distributions not sufficient for prior model.
    - Saturations are in $[0, 1]$, and may follow sharp fronts.

Estimation of large-scale covariance matrices

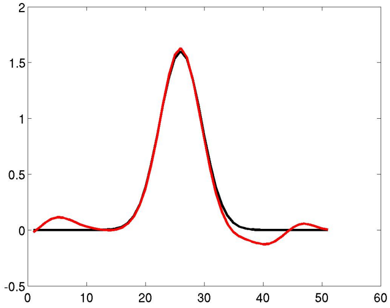
$P_k^* = \frac{1}{N-1} \sum_{i=1}^{N} (x_{k,i} - \hat{x}_k^*)(x_{k,i} - \hat{x}_k^*)^T$

Approaches:

- Localization (Houtekamer & Mitchell, 2001)
- Hierarchical ensemble filter (Anderson, 2007)
- Topic of current interest in statistics
  - Furrer & Bengtsson, 2007
  - Bickel & Levina, 2008
  - El Karoui, 2008

Linear example

- Initial model:
  - $C = [4 \cdot \exp(i - j)]_{i,j=1:51}$
  - $x \in N(0, C)$
- Measurement ($z = 2$) at $i = 26$, with $\sigma^2 = 1$
- Mean of 400 ensemble members (red)
- Analytical posterior (black)



Hierarchical ensemble filter

(Anderson, 2007, Vallès & Naevdal)
Split ensemble in N_g groups with N_e ensemble members.
Find α that minimizes the following expression (optimized over N_g Kalman gain matrices):

$$\sqrt{\sum_{l=1}^{N_g} \sum_{\substack{k=1 \\ k \neq l}}^{N_g} (\alpha\kappa_k - \kappa_l)^2}.$$

$$\alpha_{\min} = \max\left(\left\{ \left[\left(\sum_{i=1}^{N_g} \kappa_i \right)^2 \Big/ \sum_{i=1}^{N_g} \kappa_i^2 \right] - 1 \right\} \Big/ \left(N_g - 1\right), 0 \right)$$

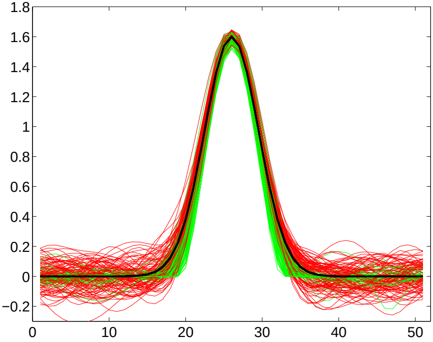
Hierarchical ensemble filter - II

A: Matrix of same size as Kalman gain K, using individual α for each entry.
Modify the updating step:

$$y^{ak,j} = y^{fk,j} + A \circ K_k \left(d^j - H y^{fk,j} \right)$$

Will show effect of $N_e = 40$, $N_g = 10$.

Hierarchical ensemble filter - III



- Green: Hierarchical ensemble filter
- Red: EnKF. Black: Analytical solution.

- Next week: “4th international workshop on ensemble Kalman filter for model updating”, see http://qp.iris.no/enkfseminar
- EnKF is a powerful tool for data assimilation.
- Norway is in the front in this area.
- More contributions from statisticians are expected!