

Updating Markov Chain Models Using the Ensemble Kalman Filter

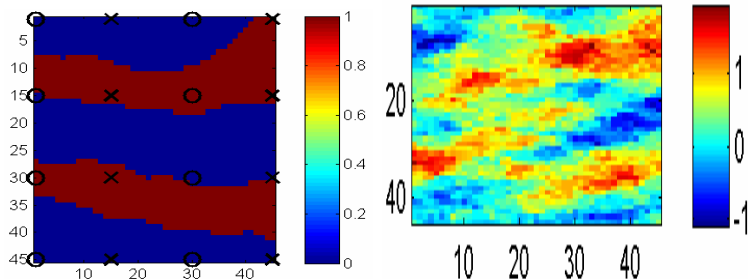
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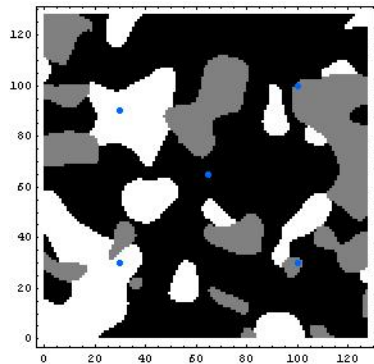
Facies models with EnKF



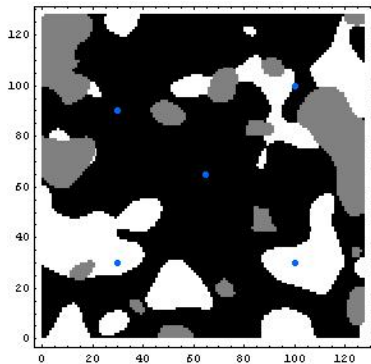
Regions of nearly uniform properties.

Illustration from Sarma, P. and W. Chen, "Generalization of the Ensemble Kalman Filter Using Kernels for Non-Gaussian Random Fields", (SPE 119177), 2009.

Facies models with EnKF



Initial model



Conditioned model

Truncated plurigaussian has been used successfully with EnKF (Liu and Oliver 2005a,b; Agbalaka and Oliver 2008; Zhao, Reynolds and Li 2008)

Why use Markov chain models?

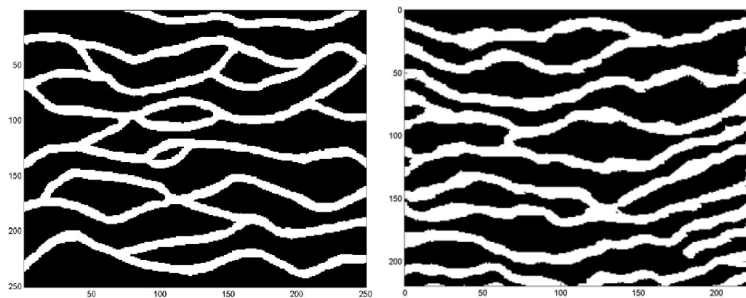


Illustration from Stien, M., H. Kjøsberg, O. Kolbjørnsen, and P. Abrahamsen, An implementation of conditional Markov mesh simulation with parameter estimation, Tech. Rep. SAND/08/04, Norwegian Computing Center, NR, 2008.

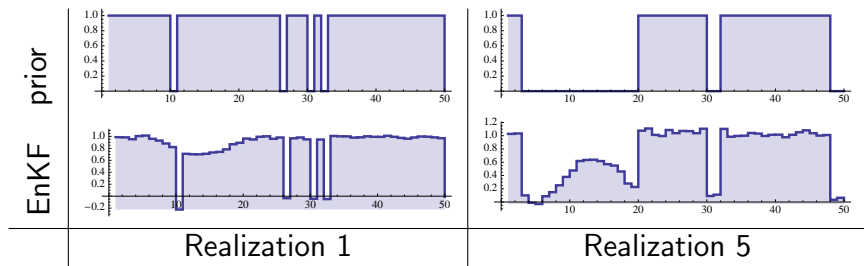
Updating Markov chain models with EnKF

1. Assimilate data using EnKF in the standard way.
2. Find the Markov chain realization that is “closest” to the analyzed model from EnKF.

Updating Markov chain models with EnKF

1. Assimilate data using EnKF in the standard way.
2. Find the Markov chain realization that is “closest” to the analyzed model from EnKF.
 - ▶ Create a set of effective observation operators.
 - ▶ Compute a vector of effective data for each realization.
 - ▶ Compute the discrete state sequence that maximizes the posteriori probability. Repeat for each Kalman update.

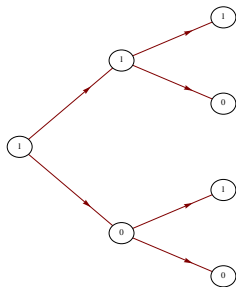
EnKF to assimilate observation of the mean



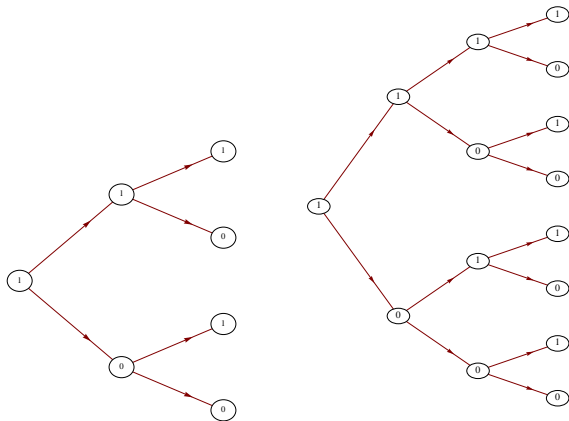
Two realizations of from Model A updated to honor the mean value over the interval 10–17. Top row shows prior realizations, bottom row shows EnKF updates. (Ensemble size 400.)

What does it mean to be close?

Most likely sequence — Viterbi algorithm

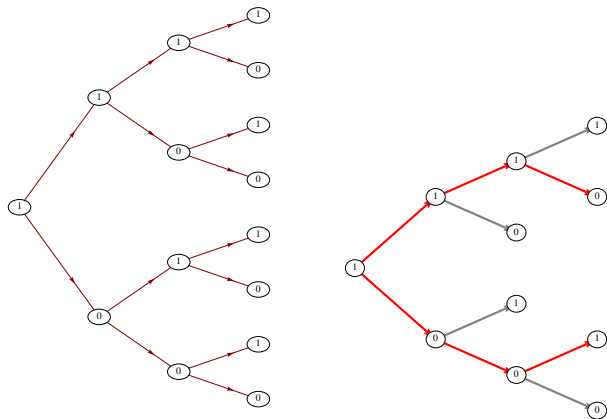


Most likely sequence — Viterbi algorithm



Number of sequences for a binary chain is 2^{L-1} .

Most likely sequence — Viterbi algorithm



Number of function evaluations required for a binary chain is $4(L - 1)$ using the Viterbi algorithm.

Likelihood required for the probability

Marginal pdf for the i th realization can be modeled as a Gaussian with mean y_i^a and covariance C_y^a .
Logarithm of the Gaussian can be rewritten

$$\begin{aligned}(y - y_i^a)^T (C_y^a)^{-1} (y - y_i^a) &= [M(y - y_i^a)]^T [M(y - y_i^a)] \\ &= [My - d_i^{\text{eff}}]^T [My - d_i^{\text{eff}}]\end{aligned}\quad (1)$$

where

$$M^T M = (C_y^a)^{-1}.$$

The effective data for the i th realization are given by

$$d_i^{\text{eff}} = M_\rho y_i^a. \quad (2)$$

Illustration of the method (Model A)

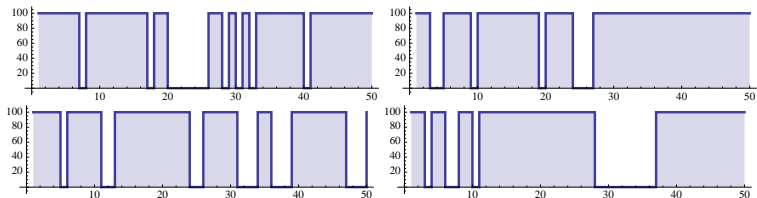
- ▶ A Markov chain with fairly long sequences of state 1 (Model A).
- ▶ This is a binary Markov chain that takes values 0.1 or 100 and whose transition matrix is

$$\mathbf{p}_A = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} = \begin{pmatrix} 0.6 & 0.4 \\ 0.1 & 0.9 \end{pmatrix}$$

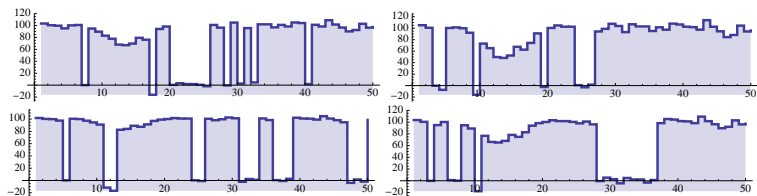
Mean duration in state 0.1 is 2.5 steps, and the mean duration in state 100 is 10.

- ▶ The ensemble contains 120 ensemble members.

Four initial ensemble members

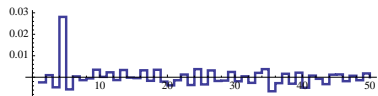


Four ensemble members after EnKF updating

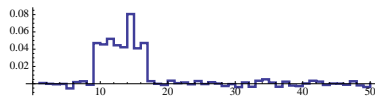


Assimilation of the observation of the mean over the interval 10 to 17.

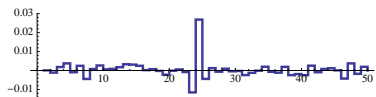
Effective observation operators ($N_e = 120$)



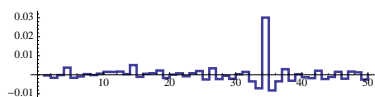
(a) $x = 5$



(b) $x = 15$



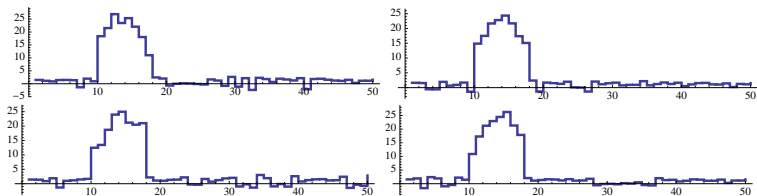
(c) $x = 25$



(d) $x = 35$

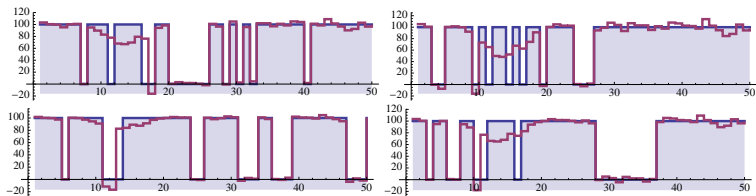
Figure: For the entire ensemble

Four realizations of effective data



Corresponding to the first four ensemble members of Model A.

First four ensemble members after Viterbi update



Compare with standard EnKF update.

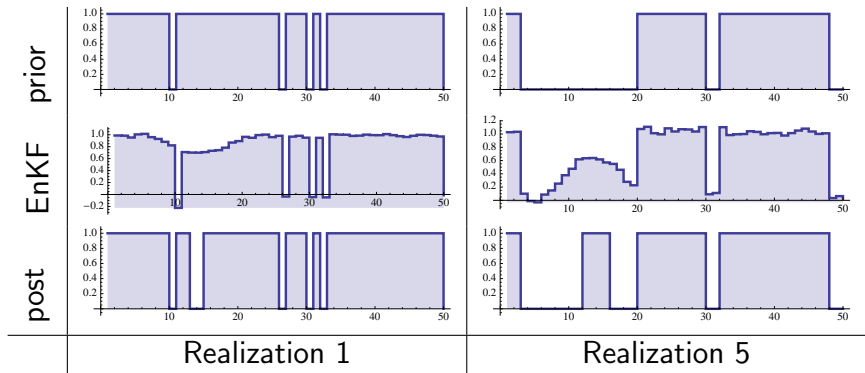


Figure: Two realizations of Markov chain Model A updated to honor the mean value over the interval 10–17.

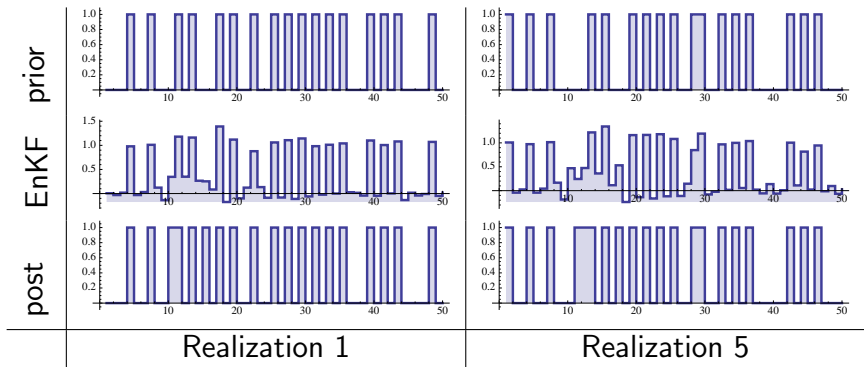


Figure: Two realizations of Markov chain Model B updated to honor the mean value over the interval 10–17.

High-order Markov chain (Model C)

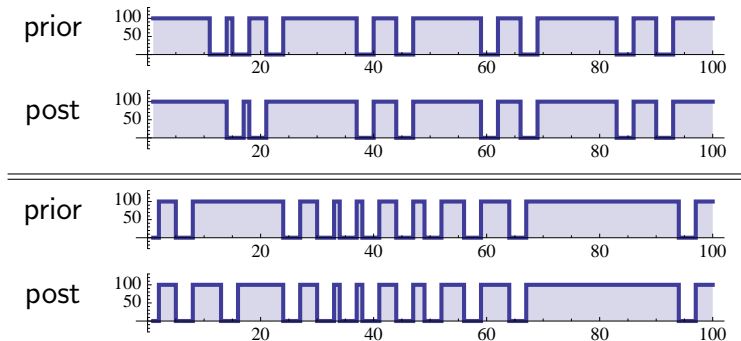


Figure: Before and after conditioning to an observation of the average permeability over the interval 10 to 17.

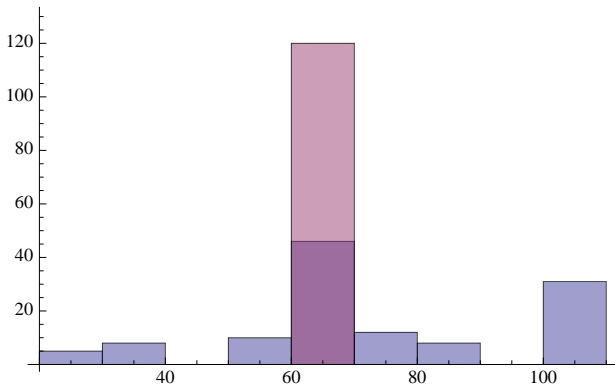
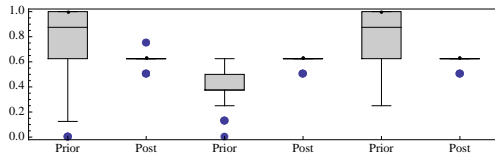


Figure: Distribution of values of predicted observations from the forecast ensemble (blue) and from the updated ensemble (pink).

Effect of ensemble size



Data predictions from the prior and Markov chain posteriori models. The observation is 0.625. First four bars correspond to $N_e = 120$, the last two bars correspond to $N_e = 40$.

Final comments

- ▶ Works well in 1D
 - ▶ assimilates data
 - ▶ honors “multipoint statistics”
- ▶ Viterbi does not extend directly to 2D, but research on 2D HMM.